Roll No. | | | | | | | | | | |

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E. / B. Tech (Full Time) - END SEMESTER ARREAR EXAMINATIONS, NOV / DEC 2024

(Common to all branches)

Semester III

MA5355 & TRANSFORM TECHNIQUES AND PARTIAL DIFFERENTIAL EQUATIONS

(Regulation 2019)

Time: 3hrs Max.Marks: 100

CO 1	To introduce the effective mathematical tools for the solutions of partial differential equations that model physical processes.
CO 2	To introduce Fourier series analysis which is central to many applications in engineering.
CO 3	To develop the analytic solutions for partial differential equations used in engineering by Fourier series.
CO 4	To acquaint the student with Fourier transform techniques used in wide variety of situations in which the functions used are not periodic.
CO 5	To develop Z- transform techniques which will perform the same task for discrete time systems as Laplace Transform, a valuable aid in analysis of continuous time systems.

BL – Bloom's Taxonomy Levels

(L1 - Remembering, L2 - Understanding, L3 - Applying, L4 - Analyzing, L5 - Evaluating, L6 - Creating)

PART- A (10 x 2 = 20 Marks)

Q. No	Questions	Marks	CO	BL
1	Solve $p+q=pq$.	2	1	L2
2	Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial z}{\partial x}=0$.	2	1	L2
3	If $x\sin x=-1-\frac{1}{2}\cos x+\pi\sin x+2\sum_{n=2}^{\infty}\frac{\cos nx}{(n-1)(n+1)}$, then find the value of $\frac{1}{3}-\frac{1}{15}+\frac{1}{35}-\cdots$.	2	2	L3
4	The Fourier series expansion of $f(x)$ in $(0,2\pi)$ is $f(x)=\sum_{n=1}^{\infty}\frac{\sin nx}{n}$. Find the root mean square value of $f(x)$ in $(0,2\pi)$.	2	2	L4
5	Classify the PDE $u_{xx}-2\sin x\ u_{xy}-\cos^2 x\ u_{yy}-\cos x\ u_y=0$.	2	3	L3
6	Solve the equation $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$, given $u(x,0)=6e^{-3x}$ by the method of separation of variables.	2	3	L3
7	Show that $F_C[f(x)\cos ax]=\frac{1}{2}[F_C(s+a)+F_C(s-a)]$.	2	4	L1
8	If $f(x)$ is an even function of x, then prove that $F(s)=F[f(x)]$ is also an even function of s.	2	4	L2
9	Find $Z\left[\frac{1}{n}\right]$.	2	5	L2
10	Form a difference equation by eliminating the arbitrary constant A from $y_n=A3^n$.	2	5	L2

PART- B (5 x 13 = 65 Marks)

Q. No	Questions	Marks	CO	BL
11 (a) (i)	Solve $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}=2e^{2x}+3x^2y$.	8	1	L5
(ii)	Form the partial differential equation by eliminating the arbitrary functions f and g from $z=f(x+iy)+g(x-iy)$.	5	1	L3
	OR			
11 (b) (i)	Solve $x(y^2+z)p+y(x^2+z)q=z(x^2-y^2)$.	8	1	L5

(ii)	Solve $yp = 2xy + \log q$.	**5**	1	L5
12 (a) (i)	Find the Fourier series up to the second harmonic from the data: $\begin{array}{c|c|c|c|c|c|c} x & 0 & \frac{\pi}{3} & \frac{2\pi}{3} & \pi & \frac{4\pi}{3} & \frac{5\pi}{3} \\ \hline y & 0.8 & 0.6 & 0.4 & 0.7 & 0.9 & 1.1 \end{array}$	**8**	2	L4
(ii)	Obtain the Fourier series for $f(x) = \begin{cases} x, & -1 < x \le 0 \\ x+2, & 0 < x < 1 \end{cases}$.	**5**	2	L3
	OR			
12 (b) (i)	Find the complex form of Fourier series of $f(x) = e^{-x}$ in $(-1,1)$.	**8**	2	L3
(ii)	Find the Fourier series for $f(x) = \lvert\cos x\rvert$ in the interval $(-\pi, \pi)$.	**5**	2	L4
13 (a) (i)	A rod of length l has its ends A and B kept at 0^oC and 100^oC respectively, until steady state condition prevail. If the temperature at B is suddenly reduced to 0^oC and maintained so, then find the temperature at a distance x from A and at any time t.	**13**	3	L6
	OR			
13 (b) (i)	A rectangular plate with insulated surfaces is 20 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge $x = 0$ is given by $u = \begin{cases} 10y, & 0 \le y \le 10 \\ 10(20-y), & 10 \le y \le 20 \end{cases}$ and the two edges as well as the other short edge are kept at 0^oC, find the steady state temperature distribution in the plate.	**13**	3	L5
14 (a) (i)	Find the Fourier transform of $f(x) = \begin{cases} 1, & \lvert x\rvert < 2 \\ 0, & \lvert x\rvert > 2 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.	**8**	4	L5
(ii)	Using Parseval's identity evaluate $\int_0^\infty \frac{x^2}{(a^2+x^2)^2} dx$.	**5**	4	L4
	OR			
14 (b) (i)	Find the Fourier transform of $e^{-a\lvert x\rvert}$ and hence evaluate $\int_0^\infty \frac{1}{(x^2+a^2)^2} dx,\ a > 0$.	**8**	4	L5
(ii)	Show that $e^{-x}\cos x = \frac{2}{\pi}\int_0^\infty \frac{\lambda^2+2}{\lambda^4+4} \cos \lambda x \, d\lambda$, using Fourier integral formula.	**5**	4	L3
15 (a) (i)	Find $Z^{-1}\left[\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right]$ by using partial fraction.	**8**	5	L5
(ii)	Find $Z[a^n \cos n\theta]$.	**5**	5	L4
	OR			
15 (b) (i)	Solve the difference equation $y(n+3) - 3y(n+1) + 2y(n) = 0$ given that $y(0) = 4$, $y(1) = 0$ and $y(2) = 8$.	**8**	5	L5
(ii)	Using convolution theorem, evaluate $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$.	**5**	5	L4

PART- C (1 x 15 = 15 Marks)

Q. No	Questions	Marks	CO	BL
16 (a) (i)	A tightly stretched string with end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If each of its points is given a velocity $\lambda x(l-x)$, then determine the displacement $y(x,t)$ at any time t.	**15**	3	L5